EQUITY, TECHNOLOGY, ASSESSMENT, AND PROBLEMATIC PLACEMENT

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EXECUTIVE SUMMARY

Introduction. This document consists of my comments on Chapters 10, 11, 12, and 13 in the second field review of the California Math Framework (CMF). These will be submitted as official "public comment" to the next revision stage; I am making them available to the public here.

Chapter 10 discusses approaches to equity in K-12 math classes, Chapter 11 treats the role of technology in math classes (including distance learning), Chapter 12 is about assessment of mathematical learning (homework, exams, etc.), and Chapter 13 concerns the criteria that creators of course material ("publishers") must meet to be approved for statewide adoption (for grades K-8).

In the 2013 CMF, the analogues of Chapters 11 and 12 were a fairly modest 10 and 13 pages respectively, while the analogues of Chapters 10 and 13 were respectively 36 and 25 pages long. Now the length of Chapter 13 on publisher criteria has become *shorter* (only 22 pages) whereas the length of Chapter 12 on assessment has exploded (a whopping 70 pages).

In the 2005 CMF, the chapter on assessment was a mere 5 pages long. What could make the discussion of assessment grow so dramatically (from 5 to 70 pages) over two CMF cycles? The primary reason is the inclusion in the current CMF of extensive opinionated discussion opposing homework and a variety of standard assessment practices. These opinionated discussions have to be removed.

The writing of Chapter 12 suggests its guidance is evidence-based, but that is false. My extensive comments elsewhere on the overall citation misrepresentation in the CMF have many entries for Chapter 12 corresponding to the opinionated discussions noted above; I won't repeat those here. The greatest concerns among Chapters 10-13 are in Chapter 12, so let me first briefly address Chapters 10, 11, and 13:

- In Chapter 10, there are proposals that disrespect the time burden on teachers and assertions that indicate a misunderstanding of the universal nature of mathematical content. It also promotes violation of the Math Placement Act in multiple places.
- In Chapter 11, the discussion of distance learning has almost nothing to do with math, and so it should all be removed. There is a separate SBE-approved May 2021 document specifically about distance learning, but its Section B on math has to be revoked due to extensive reliance on an earlier (not yet approved) CMF draft that is very far-removed from what now exists. The replacement for that Section B is where all discussion of distance learning for math should be placed. Once that is done, Chapter 11 will be half as long and its remaining concerns easy to fix.

- In Chapter 13, the concerns center around several things that occur elsewhere in the CMF: data science hype as in Chapter 5, denigration of the importance of content standards as in Chapter 1 (despite legal obligations to the contrary that are also noted in Chapter 13), and denigration of the importance of procedural skills as in Chapter 12.

Assessment concerns. Finally, we turn to Chapter 12 on assessment. The main concerns fall into three types:

(i) Irresponsible and baseless denigration of the importance of procedural fluency alongside conceptual understanding.

(ii) Advocating for the elimination of math homework and the wholesale use of subjective measures of learning (such as portfolios) in place of objective measures, thereby *reducing accountability to external authorities*.

(iii) Promoting violations of state law concerning course placement.

A sample illustration of (i) appears on lines 53-57 of Chapter 12, with the passage

> "It has long been the practice in mathematics classrooms to assess students' mathematics achievement through narrow tests of procedural knowledge. The knowledge needed for success on such tests is far from the adaptable, critical and analytical thinking needed by students in the modern world."

This promotes a cartoon view of how reliable mathematical skills for applications are acquired. It is on par with opposing that children should learn how to spell because there are spell-checkers and spelling is not part of analytical thinking.

The writing of the CMF did not involve collaboration with recognized STEM experts in industry, and its content advocacy at the high school level has caused serious concern among STEM experts in higher education who provide foundational training for jobs of the future. The CMF has to be more balanced in its discussion of the equally important fluency in procedural skills (to a non-tedious level) and conceptual understanding.

Turning to (ii), the advocacy against math homework is discussed at length in the comments on Chapter 12 in my separate compilation of citation misrepresentatons. Let us just highlight here what is said on lines 753-755:

> "*Do not include homework, if given, as any part of grading*. Homework is one of the most inequitable practices in education; its inclusion in grading adds stress to students and increases the chances of inequitable outcomes."

As public policy guidance, this is extraordinarily irresponsible. Who would ever hire an accountant that never did math homework or drive a car designed by an engineer who never did math homework?

The other component of (ii) entails *formative assessment*, a means of evaluating student progress that has some merits for flagging misunderstanding early on. I am not objecting to some use of formative assessment, but this approach is acknowledged by experts as

entailing significant new challenges upon teachers for implementation. The CMF is silent on that very practical issue.

Let us now discuss (iii). Chapter 12 omits all discussion of assessment for math course placement, and this omission is a major concern because in the time since the last CMF adoption in 2013 there has emerged a pattern of local school boards in California flagrantly violating existing state laws concerning such placement. The most visible example of this is SFUSD, and local Boards of Education in *other states* have also been engaging in such policies, following the lead of SFUSD. Hence, stopping the illegal policies in California should have effects more widely.

Passages in multiple chapters (such as 9, 10, 13) promote policies that violate the Math Placement Act. It is incumbent upon the CMF to clearly and visibly insist upon the obligations of local Boards of Education under two state laws discussed below. Since part of the illegal activity has relied on local boards creating their own placement exams that are intentionally excessively difficult (as illustrated below), the CMF has to recommend the use of third-party objective assessments when meeting the legal obligations outlined below.

The actions of SFUSD. Beginning in Fall 2014, SFUSD forced enrollment in Algebra I for the large number of 9th grade students who have already completed a UC-approved Algebra I course. This violates item (a) in §51228.2 of the California Education Code, which says districts

> "shall not assign a pupil enrolled in any of grades 9 to 12, inclusive, in a school in the school district to a course that the pupil has previously completed and received a grade determined by the school district to be sufficient to satisfy the requirements and prerequisites for admission to the California public institutions of postsecondary education and the minimum requirements for receiving a diploma of graduation from high school established in this article,"

(the law allows exceptions, but those are not applicable when the parents do not approve).

More specifically, beginning in Fall 2016, SFUSD has required such students to pass an *additional* hurdle in order to avoid retaking a course on what they have already learned: its own Math Validation Test (MVT). This test is specifically designed to be much harder than the Algebra I course, ensuring that most who attempt it will fail. This policy of an additional hurdle would be illegal even if SFUSD's MVT were not excessively difficult, though the excessive difficulty violates yet another state law as discussed below.

[Curiously, those illegal policies based on the approach of holding back students with advanced knowledge are opposite the approach of UC area-C loophole which amounts to promoting the learning of *less* math to fulfill UC admission requirements. In both cases, the people violating policies and regulations wrap themselves in the banner of equity, as if claims to be pro-equity cannot be false.]

A relevant reference on this matter is the paper (LaMar et al., 2020) cited in Chapters 2 and 12 of the CMF. The title of that paper contains the phrase "Derailing Impact

of Content standards" which directly opposes the fundamental principle of educational standards. The text of the paper confirms the illegal behavior by SFUSD: it says that opposition to the SFUSD policy came from "parents of currently accelerated students"; this admits the forced holding back of students who were already accelerated, a violation of §51228.2 as discussed above.

A report by Families for San Francisco revealed the claimed benefits from the illegal SFUSD policy to be a mirage of lies. (The SFUSD policy would have been illegal even if the claimed benefits were not fraudulent.)

Elsewhere in the California Education Code is §51224.7, the Math Placement Act that explicitly requires

> "a fair, objective, and transparent mathematics placement policy for pupils entering grade 9"

based on "objective academic measures". This forbids the use of formative assessment (that is subjective by definition, and hence prone to abuse by officials who oppose legally-mandated course placement laws), and the law mentions "statewide mathematics assessments" as an example of such a measure.

Section 1 of California Senate Bill SB-359 explicitly calls out these concerns in item (c)

> "The most egregious examples of mathematics misplacement occur with successful pupils and, disproportionately, with successful pupils of color. These successful pupils are achieving a grade of "B" or better, or are testing at proficient or even advanced proficiency on state assessments."

and two further items

> "(i) California faces a looming shortage of college-educated workers in an increasingly competitive global economy.
>
> (j) A policy for correct mathematics placement must be addressed in order to ensure a fair process and chance of success for all pupils."

illustrate the significance of adherence to this law.

Those who claim to be champions of equity should put more effort and resources into helping all students to achieve real success in learning mathematics, rather than using illegal artificial barriers, misrepresented data and citations, or fake validations to create false optics of success.

Recommendation. Chapter 12 has to fully address math course placement, and clearly state that local Boards of Education have a legal responsibility to follow the two state laws discussed above. Assessment options suitable for discussion in Chapter 12 include:

- the MDTP made freely available by UC/CSU,
- the NWEA MAP assessment,
- the iReady assessment,
- the Smarter Balanced assessment.

A broader discussion of CDE-recommended diagnostic assessment is given on the CDE webpage "Guidance on Diagnostic and Formative Assessments". The CMF also has to clearly state that in contrast, the MARS assessment (which is mentioned elsewhere in Chapter 12) is forbidden to be used for course placement purposes because it is based on *formative assessment*. That is subjective rather than objective, and hence violates the legal requirement for *objective* measures in the Math Placement Act.

1. Chapter 10

lines 36-41: In the passage

> "So, how can teachers be best supported in creating equitable and engaging mathematics learning environments for their students? Administrators and teacher leaders, such as coaches and teachers on special assignment, provide the initial, programmatic layers of support, while parents, counselors, and community members co-create an interconnected system that supports children and adolescents as they learn."

that is claiming (among other things) to discuss how to best support teachers to create an engaging environment for learning math, the CMF neglects to mention an essential feature: material that includes improved motivation for the content being taught. So here the CMF needs to call out the need for publishers to provide modern motivation for use by teachers.

lines 47-50: The passage

> "In contrast, an activity is inauthentic if students recognize it as a straightforward practice of recently-learned techniques or procedures, including the repackaging of standard exercises in forced "real-world" contexts."

properly says that the context for problems should not be artificial, but it improperly denigrates routine practice. As anyone who has watched professional athletes warm-up can attest, it is entirely fine for students to carry out some routine practice in sensible (not excessive) amounts. Dismissing the value of routine practice altogether is wrong. The tendency of the CMF to take things to extremes rather than acknowledge a balanced approach is incorrect and has to be fixed.

lines 111-115: The passage

> "Teachers perform incredibly complex work that relies on thousands of instructional decisions every day (Ball, 2018): in understanding their students' thinking, choosing tasks, deciding which questions to pose in discussion, selecting which (and whose) lines of inquiry to pursue with the class, and ensuring that all students have their authentic and culturally relevant contexts and tasks represented."

initially recognizes the difficult work of teachers, but then insists upon the standard "authentic and culturally relevant" where "authentic" would encompass the same main issue (incorporating genuine motivation). The point is not to put an additional burden on

teachers to fine-tune every lesson to a level of individualization that is impractical. A teacher cannot know what would be deemed "culturally relevant" to each student.

The primary purpose of math class is to learn math, and providing motivation that can resonate with students is a realistic goal. But the cultural variation among the class may be vast. The CMF has to bear that in mind.

lines 134-135: Here the CMF notes that one of the factors in the improvement of mathematics teaching and learning is alignment with the efforts of "College and university faculty involved in and advocating for high-quality mathematics instruction and preparation of future teachers". But nowhere does the CMF mention here the role of the college and university faculty in the STEM disciplines who will be training those students after high school for careers building on their K-12 math knowledge. These other college faculty serve an equally important role, as the MAA and NCTM have recognized in a joint statement that says

> "Faculty on both sides of the transition from secondary to college mathematics should work together to strengthen the mathematics curriculum so that students who intend to pursue a mathematically intensive career can acquire the mathematical knowledge and capabilities needed for such a career."

The CMF needs to add this group of faculty as another entry in the list of stakeholders here.

lines 198-199: In its discussion of professional training in math education, the CMF's sentence

> "More importantly, the field should prioritize professional learning opportunities that focus primarily on equity in mathematics education."

can be read as suggesting equity is not intertwined with motivation and content. A more balanced formulation is:

> "Professional learning opportunities should highlight equity alongside focus on content and motivation; each of these plays an important role in promoting improved outcomes in math classes."

The most important goal of a math class is the learning of mathematics, and efforts which support this goal for a broad spectrum of students can be an integral part of improvements to teacher effectiveness.

lines 205-211: The passage

> "Many of the stories used to define mathematics, and to talk about who does or is good at mathematics, are highly racialized and English language-centric, and are experienced that way by students (Lue and Turner, 2020). This means students' mathematics identities are shaped by social messages that are conditioned by assumptions about race and gender. Professional learning in mathematics can respond to these realities and aim for more

> than incremental change (which does little to change the framing narratives that drive inequities)."

is accusatory toward teachers, and as such as unacceptable for a public policy document. The need for students of all backgrounds to feel engaged and respected in their math classes can be discussed in more respectful ways, such as:

> "Student perception of their capacity to succeed in mathematics is shaped by messaging from teachers and society. Many efforts in recent years have focused on increasing rates of success among members of historically underrepresented groups in mathematical fields. This includes attention to role models and in professional training to pedagogical practices and materials used in the classroom."

The primary role of pedagogy is to help students to learn the mathematical *content*. The CMF should focus on the central goal of student learning of math.

This is supported by the later observations (on lines 409-410 in Chapter 10) "Professional development in any discipline has been found to be most effective when the content knowledge in that area – in this case, mathematics – is a primary focus." and (on lines 592-596)

> "In a survey of the effectiveness of 643 professional development models, only two models were found to have a significant positive effect on students' learning—lesson study and sustained content-focused summer courses with pedagogy-oriented structured academic year follow-up."

The bottom line: focus on math content.

lines 339-345: The passage

> "By taking the time to acknowledge and center contributions to mathematical understanding from Africa, South America, Asia, and indigenous peoples around the world, students can better appreciate the global nature of mathematical discovery. In a similar way, prospective teachers in methods courses can expand their understanding of teaching and learning mathematics by exploring a variety of approaches from a diverse array of cultures."

is a narrow perspective on "the global nature of mathematical discovery". The CMF should emphasize that mathematics is a *way of thinking* that is of universal nature because the mathematical formulation of scientific laws in medicine, engineering, and chemistry, as well as the logical and mathematical principles that underlie all technology, are *identical around the world* (and beyond).

Awareness of role models is valuable, and *brief* discussion of historical context for some discoveries can add an interesting human element to motivation. But the primary focus in math class has to be on learning math: there is a curriculum to be learned, and subsequent courses will build upon that knowledge.

lines 702-806: This entire vignette is promoting a company run by one of the CMF authors, and so it is a conflict of interest. It has to be removed.

In particular, in the table on line 771, the denigration of the importance of correct answers in mathematics in the "Depth over Speed" row is unacceptable. The role of reasoning alongside correct answers gives students a deeper understanding of the content (and avoids the trap of blind memorization of everything), but the CMF is wrong to closely link "correct answers" with "speed, memorization". It is subordinating the value of correct answers to "depth, creativity, visuals, and mathematical beauty" as if correct answers are of minor relevance.

On the contrary, the power of mathematics resides in the fact that it admits a stronger sense of "correct answer" than in other fields. Depth and creativity in math are important, but so are correct answers. Traditional teaching that focuses on the correct answer to the exclusion of the method is too extreme, but likewise the CMF is wrong to promote the opposite extreme. Everyone who uses autopilot, GPS, and mobile banking relies on the bedrock of correct answers in math.

lines 830-1009: Here the CMF takes up more than 7 pages on 3 vignettes about mathematical coaching. This is too much. Either write in a more focused way or cut some vignettes. If the CMF is going to take up this much space, at least have some discussion on topics beyond elementary grades, for a better balance.

lines 1010-1078: This section on **Teacher Leadership** belongs in a manual for bureaucracy, not in the CMF, so it has to be removed. It has nothing to do with math content, or even mathematics at all. Moreover, towards the end (on line 1070) it includes promotion of a company run by one of the CMF authors, a conflict of interest that has to be removed.

1130-1138: The passage

> "Partnerships with parents, families and caregivers can also provide valuable opportunities for administrators to rely upon as they work with teachers in addressing the totality of students' learning experiences. Family partnerships and experiences, especially those that are culturally and linguistically diverse, can create rich avenues of professional learning for teachers and teacher leaders. They should also draw upon teacher leaders at their school site or within their district who can provide support and knowledge of inclusive teaching approaches, especially those that focus on students who are culturally and linguistically diverse learners and students with learning differences."

is verbose word salad, a long string of buzzwords that lacks actionable guidance. For example, the discussion of "partnerships" with families is too vague to be useful, and doesn't sound realistic for over-worked teachers.

The content can be conveyed in the following short sentence that should be used instead:

> "Teaching practices should be engaging for students from many backgrounds, and attentive to students still learning English."

lines 1139-1146: The passage

> "Administrators are urged to read all of Chapter 9, especially the Introduction and the section Teaching Multidimensional Mathematics through Big Ideas and Connections, as they engage in conversations with teachers, school boards and parents on the ramifications of acceleration and tracking, and work with these same groups in careful consideration of the many alternatives which afford better access to higher level mathematics for all learners discussed in Chapter 9. In particular, Chapter 9 also elaborates on the Math Placement Act and provides a wealth of alternatives to tracking for administrators to consider."

has to be removed because it is problematic in several ways. Chapter 9 is riddled with falsified citations (as documented in my comments on citation misrepresentation throughout the CMF). In particular, Chapter 9 promotes anti-tracking and anti-acceleration guidance based on misrepresentation of cited papers.

Also, in the section of Chapter 9 highlighted here for reading, there is focus on a paper that relies on histograms of merged data from all 8th grade students (not separated according to the math course they took). That makes it impossible to justify the claim made there about de-tracking effects. There is also a dramatic claim in that part of Chapter 9 concerning a gain of "2.03 years of middle school growth" without a clear explanation for where that number comes from or why the experimental design is sufficient to justify such an assertion on the basis of the evidence shown.

lines 1145-1146: The sentence

> "In particular, Chapter 9 also elaborates on the Math Placement Act and provides a wealth of alternatives to tracking for administrators to consider."

has to be removed for two reasons: the guidance in Chapter 9 opposing acceleration is a violation of the Math Placement Act, and the "wealth of alternatives" there is largely not reliable or actionable (so it is not useful).

lines 1147-1148: The sentence

> "This framework recommends that all students take the same, rich mathematics courses in kindergarten through grade eight."

is a violation of the Math Placement Act, and therefore has to be removed. The document has to be internally consistent, and in numerous places the CMF acknowledges the need to permit acceleration for those who are ready. It is therefore impermissible for the CMF to include such sentences whose effect is to say "ignore what is written elsewhere, here is what we really think". This is not writer's discretion; the sentence must be removed.

lines 1148-1156: The passage

> "The chapters describing high school pathways and data science set out a structure for high school that will be new to many administrators, including the provision of a pathway in data science and statistics that can be taken as an alternative, or in addition, to calculus. This pathway should be open to all students, not only those who have been selected as mathematically oriented in younger grades. The provision of real data, and the encouragement of students to ask their own questions of the data, has the potential to broaden participation and make Science, Technology, Engineering, and Mathematics (STEM) pathways considerably more equitable."

has to be removed for multiple reasons. Firstly, it is referring to the MIC pathway ("pathway in data science and statistics") that has to be excised from the CMF for reasons explained at length in my comments on Chapter 8 and Appendix A (and it contradicts the claim on lines 912-913 of Chapter 8 that "MIC 3 is **not** a data science course"). Secondly, in the second sentence it appeals implicitly to acceleration (which is entirely different from math content) to promote data science hype under the false claim that the traditional and integrated pathways aren't perfectly capable of being open to all students too.

The challenge of reaching calculus in grade 12 from Algebra I in grade 9 is not addressed (and in fact is made far worse) by the MIC pathway. The final sentence of this passage is a false promise contrary to all evidence: if the aim is to diversify the STEM pipeline, the effort should be to improve motivation and engagement with the math that is most relevant to readiness for such college degrees (namely, the content of the traditional and integrated pathways), not to divert students into courses which omit significant mathematical content essential to success in such majors.

lines 1175-1178: The sentence

> "Additionally, administrators must acknowledge the inequities often perpetuated through traditional assessment strategies in the mathematics classroom, and how these assessment approaches can be re-envisioned (as described in Chapter 12) to provide a balanced approach in assessing the effectiveness of mathematics instruction."

has to be removed, because it is promoting the elimination of accountability for students to learn math content. Leave discussion of assessment and related issues to Chapter 12, rather than quoting aspects of it out of context here in Chapter 10.

This can be replaced with the sentence "Administrators are recommended to read Chapter 12 for guidance on assessment practices."

lines 1179-1180: The sentence

> "Administrators should look critically at program data to determine how systems are supporting or inhibiting access to equitable mathematics."

is not meaningful because the phrase "equitable mathematics" has no definition. Math content is of universal nature (this is the reason that cars, planes, GPS, and smartphones

work). Here the CMF seems to be conflating *equitable practices* in teaching with the actual math content. Hence, the sentence can be replaced with

> "Administrators should look at program data to determine where their district may need more attention for for students of all backgrounds to achieve success in learning mathematics."

2. CHAPTER 11

The discussion of distance learning in this chapter has almost nothing at all to do with math, and so it should all be removed. There is a separate SBE-approved May 2021 document specifically about distance learning, though its Section B on math has to be revoked due to extensive reliance on an earlier (not yet approved) CMF draft that is very far-removed from what now exists. The replacement for that Section B is where all discussion of distance learning for math should be placed.

lines 121-123: The sentence

> "This approach, focused on technological learning rather than content-area learning, has been found to be ineffective in large-scale projects."

about the ineffectiveness of technocentrist education applies to many aspects of the proposed "data science" courses. This is an additional concern about the CMF's cavalier advocacy to promote "data science" as an alernative to Algebra II (and Integrated 3) rather than (i) putting in the hard work to provide guidance on improving engagement with the latter courses, (ii) discussing data science with more honesty and transparency about its role as a high school course (not as STEM-prep).

lines 149-155: In the passage

> "As research has consistently pointed out (Reys and Arbaugh, 2001), calculator use does not hinder the learning of rich mathematics. It does hinder the learning of procedural mathematics, however, especially when that is believed to be the primary objective. In considering the use of technology, the belief that rote algorithms and procedural skills (which are easily replaced by calculators) are the most important mathematics to be learned which must be reconsidered."

there are several flaws that have to be fixed:

- The reference to (Reys and Arbaugh) is just an executive summary, not the actual NCTM document with the research being mentioned. Citations should always be to the *original research*, so the CMF should change the citation to the NCTM book *Principles and Standards of School Mathematics* published in 2000.
- In the absence of context, this passage seems to be speaking about K-12. But the reference is about grades K-5, and so that significantly affects the meaning of "does not hinder the learning of rich mathematics". So that is a tremendous inaccuracy that has to be fixed.

If this passage is to be retained, it must be very clearly stated that this is only being discussed in the context of K-5, but then there is an elephant in the room: what are the downstream effects on subsequent skill with algebra and other higher-level mathematical work when there is too much use of calculators in such early grades? This is analogous to the use of spell-checkers early on when kids are learning to write; it is a very serious issue.

- The second half of the passage denigrates the learning of basic procedural skills (an important outcome of elementary grades math, on par with learning spelling by practice and rote too). This isn't to say that rote learning can't be taken to an extreme (especially for more advanced math), but the CMF needs to *maintain a responsible balance*: procedural skills *are* very important for numerical fluency, on par with spelling in the learning of language.

  Just as spelling and vocabulary are both important in the learning of language, so it is with procedural skills and number sense for elementary grades math. The CMF must *stop the denigration of procedural skills*.

line 190: The list of courses **Integrated Math II/MIC 2** for the vignette being introduced here should be replaced with **Integrated 2/Geometry** because (i) MIC has to be eliminated from the CMF (for reasons I have explained at length in my comments on Chapter 8 and Appendix A), (ii) bias against the traditional pathway is forbidden, (iii) the topic of this vignette is well-suited to a geometry class.

line 251: The sentence "But what if there were three types of terrain, or five, or 10?" is a natural question, but it involves *multivariable optimization*, and as such is a topic far beyond the high school level (let alone for a second math course at the high school level). This is too sophisticated even with technology, due to the mathematical level of the class, and so this sentence should be removed.

lines 313-316: The passage

> "To avoid overwhelming teachers, and in deference to the multitude of knowledge and comfort levels they have, training should focus on one tool, or aspect of one system, at a time. After teachers are given opportunities to implement in their classes, then further tools can be introduced."

rightly recognizes that the use of technology has to be respectful of the time burden on teachers. But MIC 3 violates that very same principle: it is a chaotic vaguely-defined proposal that promotes extensive use of technology without clarity on the interaction with mathematics or on the pedagogical purposes of the technology. This is a moot comment since the entire MIC pathway has to be excised from the CMF, but I wanted to point out this internal inconsistency.

lines 325-327: In the passage

> "Technology use in mathematics classrooms must contribute to making the mathematics community more equitable. Thus, administrators and teachers must give special attention to issues of access when designing instructional uses of technology."

the first sentence is meaningless (technology use doesn't "make" anything equitable; it is the access to such technology which has to be equitable). The second sentence is conveying the aspect with actual content, so remove the first sentence and "Thus," at the start of the second sentence.

lines 530-908: The entire discussion of digital learning here has essentially no math content at all. Hence, it has no place in the a curricular framework for *mathematics*, and so should all be eliminated from the CMF.

To the extent the SBE wants to give guidance for digital learning in math, it should be in a replacement for Section B of the May 2021 SBE-approved document on digital learning. That Section B has to be revoked anyway, due to its extensive reliance on obsolete aspects of the prior CMF draft.

3. CHAPTER 12

lines 34-35: The sentence

> "Student mathematics assessment is evolving from rote tests of skills to multi-dimensional measures of problem-solving capacity and evidence-based reasoning."

inexcusably denigrates by the importance of procedural skills. This implies that evaluating automatic skills is less important than reasoning. But hierarchical building of skills is critical for math and most academic pursuits. For example, one could not understand this written statement without being able to read letters, link them to words and then concepts. The skill of reading involves much practice, and without this skill many levels of academic achievement suffer.

Similarly, math knowledge requires fluency in: algebraic work, mental arithmetic with small numbers (extrapolated to back-of-the-envelope calculations in all scientific work), and so on. It is therefore unacceptable for the CMF to promote the idea that assessment of mathematical knowledge does not include testing for procedural skills (in a non-tedious sense) in the elementary grades; that is the bedrock for reliable mathematical work at more advanced levels.

It is also unclear what "evidence-based" means in this CMF sentence. Hence, the sentence has to be replaced with

> "Student mathematics assessment is evolving from rote tests of skills to multi-dimensional measures of problem-solving capacity, reasoning, and procedural skills at a sensible level."

lines 38-39: In the passage

> "increasingly modern assessments continue to replace traditional tests,"

the use of the word "modern" is a judgement that is not meaningful or correct. Modern tests should follow the Standards for Educational and Psychological Testing by the American Educational Research Association, American Psychological Association, and the National Council on Measurement in Education. Tests have value if they are predictive, reliable, and demonstrated to identify strengths and weaknesses that guide learning to lead to improved achievement or sense of self-efficacy.

lines 51-52: The passage

> "Important mathematics learning is multidimensional and can be demonstrated through many forms of communication . . . "

should have the buzzword "is multidimensional" removed (it is not adding anything to this sense), and "and can" replaced with "often can".

lines 53-57: The passage

> "It has long been the practice in mathematics classrooms to assess students' mathematics achievement through narrow tests of procedural knowledge. The knowledge needed for success on such tests is far from the adaptable, critical and analytical thinking needed by students in the modern world."

persists in the irresponsible and baseless denigration of procedural fluency alongside conceptual knowledge, and the promotion of a cartoon view of how reliable mathematical skills for applications are acquired. It is on par with opposing that children learn how to spell.

The writing of the CMF did not involve consultation with recognized STEM experts in industry, and its content advocacy at the high school level is widely opposed by the STEM experts in higher education who provide foundational training for jobs of the future. Hence, the second sentence in this passage has to be removed, and overall the CMF has to be more balanced in its discussion of the equally important fluency in procedural skills (to a non-tedious level) and conceptual understanding. The devaluing of procedural skills in math has to be eliminated from the document.

lines 60-62: The passage

> "decontextualized questions that require procedural knowledge, without any reasoning or problem solving, have been found to be of limited use in predicting success in college and the workplace."

suggests that problems without "context" do not involve reasoning and do not predict future academic success. No evidence is given for this claim.

lines 68-69: The passage "Recommendations for assessment equitable teaching and assessing . . . " is ungrammatical and so has to be fixed (it is unclear what is intended here).

lines 74-79: In the passage

> "A particularly damaging assessment practice to avoid is the use of timed tests to assess speed of mathematical fact retention, as such tests have been

> found to prompt mathematics anxiety. When anxious, the working memory—the art of the brain needed for reproducing mathematics facts—is compromised. Math anxiety has now been recorded in students as young as five years old (Ramirez et al., 2013) and timed tests are a major cause of this debilitating, often life-long condition (Boaler, 2014)."

the first sentence is yet another instance of the CMF objecting to one extreme (perhaps excessive time-pressure tests) and going to the other (objecting to time tests for fact retention at all). The ability to reproduce certain basic mathematical facts (e.g., single-digit sums and single-digit products) without having to think about it *is an important skill* much as success at typing or playing the piano requires instinctive awareness of the location of the keys. Studies about math anxiety are not a basis for denial of reality about the foundational skills for later mathematical knowledge.

lines 83-86: The passage

> "Alternative activities can be used that develop mathematics fact fluency through engaging, conceptual, visual activities, instead of anxiety producing, speed tests. Inflexible, narrow methods of assessing mathematical competence also disadvantage students with learning differences."

is flawed in two ways. First, timed tests are not a method for *developing* fact fluency: they are a means of checking it. So the first sentence is putting the cart before the horse. By all means a teacher should seek a variety of ways to help kids to acquire fact fluency, but at the end of the day a sensibly-timed test is reasonable given the *very meaning* of "fact fluency".

Furthermore, the fact that some students with learning differences may require an alternative form of assessment for fact fluency is not a sensible basis for declaring that nobody else can be assessed in a specific way.

lines 88-91: The sentence

> "In mathematics, assessments should be flexible, allowing for multiple means of expression, such as talking, writing words, drawing using manipulatives or typing responses, as well as provide actionable feedback to students.'

has to be removed because sometimes *process matters*, and overburdened teachers can't be reasonably expected to offer a wide array of different types of assessments.

line 99: The meaning of the phrase "Assessments should match the focus on big ideas" doesn't make sense: the CMF is once more going from one extreme (the common exclusive focus in the past on rote methods, disregarding reasoning) to another (only assessing "big ideas" knowledge). This dichotomy is false, and promotes the denial of the reality of the importance of procedural knowledge as part of mathematical learning.

line 106: In Figure 12.1, there are far too many difference sizes for the circles. See my comments for line 613 in Chapter 1 for more about this issue.

lines 117-123: It is unclear here what "the standard" on lines 117 and 122 means in connection with the huge table below line 122. Is it an SMP (Standard of Mathematical

Practice) as in the 3rd column, or an "indicator" as in the 4th column? In this table, "Commmunicating Stories with Data" has to be replaced with "Reasoning with Data" everywhere that it appears.

The abbreviation "TBT" for "to be completed by the teacher" is too obscure (its definition on line 116 is too implicit to be recognized). It is furthermore unclear what a teacher is meant to enter in the right two columns of the huge table here: numbers in some range, or what? Is it measuring a mixture of SMP's and indicator skills? This is rather unclear. For example, in the second row of the huge table, there are three SMP's and one indicator; what is a teacher supposed to do here?

lines 133-155: There is far too little actionable guidance on how to implement formative assessment here. It is all rather opaque and the references given *have nothing to do with math*, so there is not any evident relevance to K-12 math classrooms on the basis of what is written here. Thus, this passage should be removed unless all references can be replaced with ones visibly relevant to math.

The feature called "Evidence of learning" is not accompanied by any examples (so it will be unclear to teachers why type of evidence to seek). The feature called "A collaborative classroom culture" doesn't make sense in the context of assessment: assess each student on how collaborative the class is?

This guidance is sorely lacking in clarity about the sustainability of formative assessment. Moreover, the CMF is silent on the essential question of whether publishers are expected to provide templates for formative assessment or if this is yet another imposition on overworked teachers.

Two reasons that the list of features of formation assessment here is perplexing in the context of K-12 mathematics are:

- the original reference (Linquanti, 2014) from which it is all plagiarized specifically says that *formative assessment is for college or career-ready standards*.
- the other reference from which it has been lifted is a CDE document on *English Language Arts*.

Neither of those is related to mathematics in K-12. The reference on lines 154-155 for the definition of formative assessment is a document of generic advice by Regents of the University of California that doesn't have anything tailored to math (as far as I can see).

The sentence

> "Ongoing research and evidence on formative assessment illustrates how it improves student learning in time to achieve intended instructional outcomes."

is referenced to a document *on English learning*.

So to sum up, the CMF provides zero references about formative assessment related to K-12 math, and yet is recommending this as statewide policy. That doesn't make any sense. Where is *relevant* literature on this for K-12 math?

lines 166-169: As part of its guidance for teachers to offer students multiple means of assessment, the CMF says:

> "An illustration of this can be as simple as allowing students the option to talk through their solution by pointing and verbalizing (instead of requiring writing), or using arrows and circles to highlight particular pieces of evidence in their solution rather than repeating statements in their explanation."

This advice seems to be disconnected from actual experience teaching K-12 math: teachers have so much on their plate, and the CMF is casually proposing that individual students be allowed to submit "solutions" to math work without doing any writing, rather pointing and verbalizing. How is this at all realistic in an actual classroom for math?

Part of mathematical learning is standard forms of communication, much as students in an English class are expected to learn how to write essays adhering to a standard structure. So this should all be removed.

lines 171-175: The passage

> "Aligning assessment with one or more UDL principles can better inform the teacher of what students are learning, and multiple means of representation, whether used to inform formative assessment of daily progress or as a summative display of enduring mathematical understanding, can create a complex and diverse mosaic of student achievement."

seems disconnected from the reality of K-12 classrooms. What is "a complex and diverse mosaic of student achievement"? Teachers have only a finite amount of time in the day, and a curriculum to cover. They cannot be expected to systematically design multiple forms of assessment in their classes. It seems insulting to teachers for the CMF to makes such proposals without any actionable guidance on sustainable implementation. This should all be removed.

lines 179-184: The passage

> "shift in assessment practices which has distinct benefits for students being able to show their understanding in alternative ways. For example, students can video record their thinking related to a task or they can post answers in a live chat or anonymous poll. By considering and planning for the variety of ways in which students can demonstrate their skills and knowledge, they are better able to provide teachers with the information on what they succeed in doing, and where their challenges are."

seems disconnected from the reality of K-12 classrooms. Student submission of videos to record their thinking? Practices used during the pandemic were exhausting for teachers, and although some may have utility into the future it is not sustainable to impose upon teachers that each opportunity for student assessment should be treated like choose-your-own-adventure for each student.

lines 187-196: In Figure 12.2, the CMF gives a table copied from a document on assessment in English classes (it is Figure 8.3 in the cited reference). It is unclear what a K-12 math teacher is supposed to do with this table. The mention of self-assessment on line 195, while perhaps sensible for some purposes in an English class, doesn't make sense in a math class and so further illustrates how perplexing the CMF's assessment guidance is here.

Where is the evidence that this guidance is meaningful and useful for K-12 *math* classes? This should all be removed.

lines 197-201: Here we have Figure 12.3 that is exactly Figure 8.4 in the cited document that is about English classes. It is entirely unclear what the picture here is supposed to mean. What is an actual K-12 math teacher supposed to do with any of this? It makes no sense, and so should be removed.

lines 202-208: These three pages of tables have *nothing to do with math* and so have to be removed. This is all lifted from Figure 8.5 in the cited reference about English classes. It should be removed from the CMF due to incoherence for K-12 math classes.

lines 210-212: The CMF says

> "Formative assessment is the collection of evidence to provide day-to-day feedback to students and teachers, so that teachers can adapt their instruction and students become self-aware learners who take responsibility for their learning."

but *how is a math teacher supposed to make sense of this*? What does it mean for students in a K-12 math class to be "self-aware learners who take responsibility for their learning"? Buzzwords have to be replaced with precise actionable guidance.

lines 215-217: The sentence

> "There are a number of aspects to effective formative assessment including embedded formative assessment, rubrics, teacher diagnostic comments, self and peer assessment."

is word salad and so should be removed.

lines 263-264: The CMF says here that formative assessment lessons rely upon:

> "the implementation of multi-dimensional, group-worthy tasks, which are challenging, open-ended, and require a range of ways of working."

but how is a teacher supposed to *find* such tasks? And why must tasks be chosen to avoid allowing a kid who has mastered the subject to simply know how to solve it in a standard way? Are publishers being tasked by the CMF with developing such material? This is all rather vaguely phrased.

lines 265-267: The sentence

> "As teachers work to manage heterogeneous groupwork and assign competence they will encounter opportunities to listen to student thinking and to assess formatively."

has to be removed since at the high school level it can be used to advocate for anti-acceleration that is a violation of the Math Placement Act.

lines 274-293: Is this vignette about a teacher named Vince an actual true story? It is unclear what to make of the sentence

> "Vince said that the discussions gave him the best information he had ever had on his students' understanding of the mathematics he was teaching."

This is such an extreme statement that it is hard to take seriously.

Early in this passage, the CMF says that Vince

> "wondered if the summative assessments he had been using could be used in a formative manner."

But when using an assessment in a way contrary to its design purpose, how can one know what a student can do independently? A better course of action is to perform summative assessments so the teaching knows what the student can do and then test limits by asking about the thought process. That enables the teacher to learn how a student performs independently first and then to understand where they have limitations. For example if a skill is not automatic then a student might make careless errors but understand the problem. Such feedback is important.

lines 294-295: The CMF says

> "A rich repository of free lessons supporting teachers in formative assessment are the Classroom Challenges housed at the Mathematics Assessment Resource Service (MARS)."

yet no actual reference for the resource is given. How is a teacher supposed to find this?

line 296: Here at long last the CMF says that the formative assessment is done via active learning. This is the first concrete statement I found after very many pages on formative assessment where the reader is explicitly told that the method entails active learning. If so, this should have been said and discussed in detail much earlier.

line 339: The CMF speaks of "Rubrics that are outcomes-based, as opposed to skill-specific,", but what do "outcomes-based" and "skill-specific" mean?

lines 345-346:

lines 351-352: In the huge table on pages 23 and 24, there are several aspects that are perplexing for math classes (and seem likely to be very time-consuming for teachers).

On page 23, the right column for Mastery includes the feature

> "My work includes a reflection of how I monitored myself while I was working and adjusted my plan when necessary."

This is very strange: mastery of content in a math class certainly shouldn't include self-reflection on the solution process. A student who dives in and solves the problem in a conventional manner is demonstrating mastery perfectly well. There is no pedagogical reason that mastering a topic in math has to require self-reflection.

This isn't to say that students should be discouraged from giving their reasoning, or that a kid who is stuck should be discouraged from writing out what they have tried. But mastery certainly does not require essay-writing.

Likewise, in the 3rd, 4th, and 5th columns in the table on page 24, the emphasis on interpreting everything in terms of "real-life contexts" is myopic. Of course it is valuable for kids to have a sense of motivation for the procedures and methods they learn in a math class, but when practicing core skills it is not at all incumbent upon the student to interpret everything they do with a real-life example (e.g., the kid may simply be practicing the process to solve two linear equations in two unknowns, in which case what matters is the skill in algebra).

In fact, part of the power of mathematics is its capacity to *unify* methods from disparate contexts within a single framework. For such situations, putting everything in terms of a real-world context can be *distracting* from practice with the method. Moreover, sometimes a mathematical method for solving a physical problem involves intermediate steps with *no physical interpretation*.

To sum up: the CMF has to drop its unrealistic emphasis that the practice of mathematical procedures should always be done with real-world interpretation for the work.

lines 353-375: This discussion of a math performance assessment rubric is full of judgements rather than empirically-measured evaluations. The CMF has to explain what a student is meant to do with such feedback to gauge their level of mastery and to identify where they need to do more work for improved learning.

line 436: In the picture here, it is suggested that the highest level of making sense of problems and persevering in solving them entails finding 2 or 3 different methods of solution. This is flat-out wrong. Sometimes what matters is to learn a specific method, and there may be only one sensible way to solve a specific problem. Certainly it may occur that multiple solution methods are available, but this is not at all a necessity (and to suggest otherwise is a denial of mathematical reality).

line 454: The end of the line of text here is missing the inequality: insert "$x + 98 > 150$".

lines 461-466: The sample responses here are not really solutions: they amount to simply proclaiming an answer out of thin air, and then restating that as a second sentence. How can the CMF be offering up such empty solutions as its only sample responses for assessment purposes?

line 512: In this vignette taking place in a kindergarten class, it is said here that a group of kids arranges 261 different measurements of silkworm lengths into a histogram. This is an astoundingly unrealistic amount of time sunk into a tedious activity. Moreover, measuring length in standard units is above kindergarten level, and the counting involved in some of these tabulations is rather beyond the level of a kindergarten class. The CMF has to provide *realistic* and time-efficient vignettes.

line 558: The second row of the table at the top of page 34 has to be removed, since "I listened to diverse views and incorporated these ideas into my work" is irrelevant to

math. Certainly it is instructive for students to hear from peers about alternative approaches to solving a problem, but this is irrelevant to assessment rubrics for mathematical knowledge.

lines 576-580: The passage

> "If students appear to have understood content before it is taught or at an early stage, they will be helped by providing additional opportunities for productive struggle and deeper, more innovative problem solving through investigative tasks. All students in a class can be given opportunities for appropriate struggle and challenge if open-ended investigative tasks are used."

gives the bizarre guidance that a student who already knows the material should be told they do not know it deeply, and has to be forced to struggle with it. This is Potemkin education, artificially wasting the student's time.

lines 591-597: The sentence "Different researchers have compared the impact of grades with diagnostic feedback" is setting up a false choice. Both devices serve a useful role, so there is no reason for the CMF to be proposing that teachers have to choose between only using grades or only using diagnostic feedback. For example, in the experiment discussed here, half of a class received grades and half received diagnostic feedback without a grade, but why not the entirely normal third option: a grade *with* diagnostic feedback? Why does the CMF not discuss such a middle ground between the two extremes?

lines 619-620: The sentence

> "A teacher giving comments to students once a week is more useful than frequent grades and test scores"

has to be removed: it is made with no evidence, and no discussion of obvious alternatives (grades without being "frequent", accompnied by feedback). This reads like ideology rather than disinterested pedagogical guidance.

The CMF is promoting a mechanism to eliminate systematic accountability to an external authority, and it is not at all appropriate for the CMF to be delving into such matters. It is such digression far beyond the purview of the CMF that causes this chapter to be so astoundingly long compared to the previous CMF.

lines 678-701: Within the proposed self-assessment for Algebra I, on the topic of Understanding Functions the CMF proposes as a standard

- I can decode function notation and explain how the output of a function is matched to its input.

Setting aside that "matched to" is somewhat obscure wording, a more basic problem here is that function notation $f(x)$ is an Algebra II topic, not Algebra I.

Also, on lines 682-683, it is said that "peer assessment" involves "giving students clear criteria for assessment" yet *no examples are given*. Why should students be spending class time evaluating their peers? There is a curriculum to be learned, and grading-type work

is part of what a teacher is paid to do. The CMF needs to address this in detail with references. Sentences such as

> "Some people refer to this as inviting students into the guild—giving students the powerful knowledge they perceive only teachers to hold—which empowers them to take charge of their learning."

don't seem at all appropriate for the CMF (given its official purpose); this should be removed.

lines 705-710: Here the CMF is proposing a sense of mastery that is decoupled from accountability for learning the actual content standards. This is not acceptable, and so this has to be rewritten to affirm the responsibility for learning the content standards, no matter the method of assessment.

lines 790-801: This list of topics under Kindergarten Mathematics is chaotic: a list of topics in music, physical education, science, and social studies. what is appearing in the text? This has to all be removed, since it is nonsensical in guidance on mastery-based grading for *math*.

lines 807–823: With a proposed list of topics for mastery-based grading in Grade 6 Math, there are two instances of exact duplication of multiple items, as if the assembly of the list was done by blind cut-and-pasting: lines 816-820 identically repeat lines 807-811, and lines 821-823 identically repeat lines 812-814. Such duplication has to be removed.

line 890: In the table of learning targets LT1–LT10, the last three items LT8, LT9, and LT10 are devoid of pedagogically meaningful mathematical substance (e.g. "Agency, Ownership, and Identity" have nothing to do with success at learning the content standards, which is the essential purpose of a math class). Hence, these last three entries have to be removed.

lines 909-913: The table of outcomes here for a *Final Academic Grade* gives as the options only A, B, C, D, F – no $A-$ or $B+$ etc. This is too harsh, having no leeway for students who are reasonably between different levels of performance. The table has to be expanded in order to be realistic.

On lines 910-913, the passage

> "One key benefit of using mastery-based grading is that it includes a lot more information on what students actually know. When it includes opportunities for reassessment, and students working with feedback to improve their results, it also encourages important growth mindset messages."

reads like a recipe for chaos in a real K-12 math classroom: teachers often don't have time to be regrading submissions of all student work (as will happen a lot when the option of a higher grade is dangled in front of students).

lines 931-937: The passage

> "In traditional grading systems, points are often offered for participation, attendance, behavior and homework completion. These measures often

> bring inequity into the grading system as students outside circumstances will impact these aspects of their grade. The final grade becomes more about behaviors than learning. While mastery grading is not a panacea to fix inequities in assessments, it ensures grades and assessment relate to demonstrated knowledge rather than behaviors that may not reflect student's actual learning."

is accompanied by no evidence; it is just declaring that mastery-based grading is a magical method whereas traditional grading is something awful. This is reminiscent of how the CMF discusses Algebra II in comparison to data science. This is not evidence-based disinterested public policy guidance; it is ideology.

lines 976-978: The passage

> "As teachers continue to collect formative data about students' language, they can act on that data by assessing growth over time, adjust instruction, and consider possible flexible groupings to provide more targeted support."

seems to be proposing a lot of extra work for each individual teacher. Where is the evidence that this is realistic or sustainable?

lines 1019-1024: As an example of a summative assessment question at the middle school level, here the CMF offers something that is far below the middle school level (determining the point 8 units to the right of -17 on a number line). This discussion of "reasoning fully" for a question of this type is going to confuse students (reasoning about the calculation $-17 + 8 = -9$).

lines 1034-1040: The passage

> "Some teachers believe that if learners can retake and get full marks on their second attempt, it encourages students to take initial assessments less seriously, but this is not how students approach such opportunities. Allowing students to retake work sends an important growth mindset message and encourages further learning. Just as career mathematicians are constantly revising their work and conjectures, students should be allowed the same fluidity in their own learning process."

makes multiple assertions with no evidence, and certainly raises significant practical concerns: it's one thing to resubmit an essay in an English class, but to resubmit work on an exam for a math class either entails no prompt dissemination of solutions (which can be pedagogically problematic) or else students simply copying from such solutions. Furthermore, in the real world of busy teachers why would there be *time* for such resubmissions on a regular basis? Again, the absence of cited evidence is a serious concern.

The analogy with work of mathematicians is an apples and oranges comparison, and so has to be removed.

lines 1044-1073: This vignette on resubmission of work in a math class is an invented story with no cited evidence. It is a fantasyland with no basis in reality, and as such is not

meaningful guidance for anything. Since the entire discussion is untethered to evidence, the CMF can make up whatever feel-good story fits the promotion of its pre-determined preferences.

The analogy proposed on line 1048 between English classes and math classes for resubmitted work is glossing over many evident huge differences in the nature of resubmitted work for such classes.

lines 1074-1099: This discussion promoting the wholesale reliance on portfolios in a math class is another evidence-free series of opinions and ideology about assessment, the main upshot of which will be to undermine accountability to an external authority. It has to be removed.

To illustrate how disconnected from reality this is, consider the passage

> "Perhaps the most comprehensive way to assess student learning is through a portfolio – a collection of work that communicates students' activities over a length of time."

A portfolio is *a way*, but "the most comprehensive"? On what basis is such a claim being made?

For example, why is the assessment of learning in trigonometry well-suited to a portfolio (whatever that would mean for trigonometry), rather than a conventional assessment based on solving trigonometry problems (which is what matters for using the trigonometry to do other things)?

The later sentence

> "Portfolios are particularly appropriate ways of assessing data science projects."

is a self-fulfilling prophecy: of course for *projects* a portfolio may be more sensible, but why should data science be specifically centered around projects? Problem sets are an entirely standard means of submitting work in computer science and statistics classes, and likewise for data science.

There is nothing special about portfolios, and for most topics in math it is important to get timely feedback on work, so using portfolios doesn't make sense. Even for data science there is no reason given for the claims being made. Overall, the discussion of portfolios seems disconnected from the reality of learning math skills at the high school level.

4. CHAPTER 13

lines 61-63: Here the CMF rightly states

> "Instructional materials for mathematics in California should place a strong emphasis on students' engagement in mathematics in the ways described in the CA CCSSM"

yet this strong emphasis is violated by some of the CMF's discussions about MIC 3. This is one of many reasons why all discussion of MIC has to be excised from the entire document.

lines 72-74: The passage

> "The principle of focus is closely tied to the goal of depth of understanding. The principle derives from a need to confront the mile-wide but inch-deep mathematics curriculum experienced by many."

correctly highlights the need for math teaching to include conceptual understanding (to a reasonable degree) alongside procedural skills, but its "mile-wide but inch-deep mathematics curriculum" criticism applies to the proposed data science courses (e.g., the advanced data science course described in Chapter 5, and the Introduction to Data Science course from UCLA that is promoted in multiples places in the CMF).

Indeed, I have heard teachers of such data science courses refer to them in such terms. This is one of the reasons that the CMF's treatment of high school data science is seriously flawed, and can only be fixed responsibly by bringing in recognized and *disinterested* experts to collaborate with high school teachers to write a new appendix on data science guidance that replaces Chapter 5 (which has to be removed from the document, for reasons explained at length in my comments on Chapter 5).

lines 100-110: Here the CMF is promoting the technically correct but in fact very misleading statement that local education agencies "are not required to purchase state-adopted instructional materials" for grades K-8, and noting that such agencies have the "sole responsibility and authority" for adoption of material at the high school level.

This is disingenuous because in practice (as I have been told by people with extensive experience in California math education) many such agencies, especially under-resourced school districts, do follow the CMF guidance: they purchase state-approved material for K-8, and follow CMF recommendations for high school. Indeed, this provides the most secure legal cover for district policies.

All claims (such as I have seen repeatedly in media discussion of the CMF) that the CMF recommendations and guidance don't matter, on the grounds that districts can do as they wish, are an evasion of the CMF's fundamental responsibility to those who fully rely on the public schools.

Therefore the passage in lines 100-110 has to be expanded to acknowledge that the CMF guidance *does matter* and that normally most school districts *do follow* its guidance. That would properly recognize the responsibility of all CMF writers, CFCC members, math IQC members, and SBE members who will vote on whether to approve it.

The public has a right to know where the buck stops, and it has to be acknowledged here. This is policy affecting the education and future lives of millions of kids.

lines 168-171, 184-187: As the first criterion for evaluation of course resource for K-8 submitted for state approval, the CMF rightly calls out the fundamental role of the SBE-approved content standards:

> "1. **Mathematics Content/Alignment with the Standards**. Content as specified in the CA CCSSM, including the SMPs, and sequence and organization of the mathematics program that provide structure for what students should learn at each grade level."

and then on lines 184-187 it is further emphasized with the passage

> "Materials that fail to meet all of the criteria in category 1 (Mathematics Content/Alignment with the Standards) will not be considered suitable for adoption. The criteria for category 1 must be met in the core materials or via the primary means of instruction, rather than in ancillary components."

This demonstrates unambiguously the *falseness* of the statement on lines 438-439 of Chapter 1 "that publishers and teachers avoid organizing around the detailed content standards".

This also shows that the MIC pathway has to be removed, since MIC 3 fails this criterion (based on the chaotic information about it in Chapter 8 and Appendix A) and MIC 1 and MIC 2 are merely carbon-copies of the existing Integrated 1 and 2.

lines 241-243: The sentence

> "The *Mathematics Framework* recommends that all students take grade-level content and that students who are advanced have opportunities to extend ideas and work in more depth."

is endorsing violation of the Math Placement Act and so has to be removed.

lines 263-264: The publisher adoption criterion

> "All standards must be listed in their entirety with their cluster heading included."

is a criterion that the CMF itself fails to follow in its own treatment of the content standards. This is seen throughout Appendix A for high school standards in the first two years, and other chapters for lower grades.

Exactly as this criterion is applied to providers of instructional material for state adoption, so must it be followed by the CMF too.

lines 329-330: The statement here that instructional resources should "make learning more equitable for all people based on scientific insights into how humans learn" sounds like the neuroscience pseudo-science from Chapter 1. Hence, it should be removed unless there is much greater clarity about what "scientific insights" means and why it is reliable in this setting.

lines 342-343: The adoption requirement

> "5. Teacher resources include guidance on and references to the "big ideas" of mathematics, consistent with the 2022 Mathematics Framework."

should be removed: the "big ideas" choices in the CMF are an expository device by the CMF writers without evidence-based justification, and so they do not rise to the level of significance to be imposed on teacher resources.

lines 370-371: The adoption requirement that teacher resources should give guidance on

> "identifying areas where data science is woven into content and activities, consistent with Chapter 5 of the 2022 Mathematics Framework"

has to be removed for multiple reasons:

- Chapter 5 (which is mentioned in an essential way) has to be removed, for reasons explained at length in my comments on Chapter 5.
- It is a tremendously impractical burden on publishers to have to adhere to this entirely arbitrary rule that has been pulled out of thin air: why does "data science" require such special treatment whereas other areas of math (e.g., algebra) do not?
- There is a significant conflict of interest due to one of the CMF writers rolling out their own data science course during the writing of this document, and furthermore coordinating with the lead author of the other prominent data science course on their expansion plans. This adoption requirement is essentially compelling all publishers to engage in activities promoting the spread of data science education.

lines 437-443: Here the CMF speaks of promoting "equitable learning experiences" and "equitable outcomes" for good teaching. These phrases have to be defined, for clarity. On lines 443-444 the CMF says that "Publishers should consider the lens of equity as discussed in the [CMF] when developing lessons and units for instructional material"; the phrase "lens of equity" has to be defined.

lines 464, 472-473: In these lines which discuss content depth and what makes content meaningful and important, the CMF has to explicitly emphasize that publishers are expected to provide engaging contemporary motivational contexts for the introduction of topics in their material.

lines 515-517: The passage

> "In contrast, an activity is inauthentic if students recognize it as a straightforward practice of recently-learned techniques or procedures,"

is denigrating the practice of basic skills. That is unacceptable in a document guiding public education policy in mathematics. Certainly students should not be given excessive amounts of routine practice, but a sensible amount of such practice is as essential to attaining fluency in mathematics as warm-up is for athletes and playing scales are for piano students. Hence, this passage has to be removed.