Mathematics Framework FAQs

Information and frequently asked questions about the draft *Mathematics Framework*.

Updated July 5, 2023

Citations included in these FAQs can be found in appendix B of the *Mathematics Framework*.

What is the purpose of the framework?

Curriculum frameworks offer guidance for implementing content standards. Frameworks describe the curriculum and instruction necessary to help students achieve proficiency, and they specify the design of instructional materials and professional development. Further, they provide guidelines and selected research-based approaches for implementing instruction to ensure optimal benefits for all students. Additional information is available on the Curriculum Frameworks and Instructional Materials web page.

The 2023 *California Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve* (*Mathematics Framework*) will help local educators as they work to diversify options and improve math outcomes for all students, including those who excel and those who struggle to successfully access math content and skills.

Why is the *Mathematics Framework* being revised?

The California State Board of Education (SBE) adopts curriculum frameworks for kindergarten through grade twelve (K–12) in accordance with California *Education Code (EC)* Section 51002, which calls for the development of "broad minimum standards and guidelines for educational programs." Curriculum frameworks for English language arts/English language development, mathematics, history–social science, science, and other content areas are aligned to the SBE-adopted academic content standards. The SBE adopted the current *Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve* in November 2013. *EC* Section 60200(b)(1) directs the IQC to develop and the SBE to adopt a revised *Mathematics Framework*, which will occur in 2023.

Nationally, California's mathematics achievement ranks below the national average and, like most other states, experienced declines between 2019 and 2022 on the National Assessment of Educational Progress (NAEP) fourth and eighth grade math assessments. On California's most recently reported Smarter Balanced assessment, only 33 percent of students met or exceeded math proficiency standards in 2022, and substantial achievement gaps exist, especially for Black,

American Indian or Alaska Native, and Latino students. California students' demonstration of deep mathematical learning on local and state assessments continues to be a concern and a priority for districts. This includes the importance of high levels of mathematics understanding for college and career preparedness.

To address the urgent need to improve mathematics learning, the framework builds on learning from almost 10 years of implementing the 2013 California Common Core State Standards for Mathematics (CA CCSSM) and provides new guidance to jump-start and accelerate learning. Helping students develop the thinking skills and content knowledge necessary to master the standards is critical to improving academic performance and creating conditions for later success. This revision will update the guidance for teachers, administrators, and other educational decision-makers to ensure effective implementation of the CA CCSSM.

What is the process for framework development?

The guidance contained in the draft *Mathematics Framework* was informed by the contributions of members of the public, individuals, and interested organizations and coalitions of organizations, who provided feedback and numerous recommendations to the Mathematics Curriculum Framework and Evaluation Criteria Committee (CFCC) and the Instructional Quality Commission (IQC). Every advisory committee meeting, including those of the IQC, was publicly-noticed, open to the public, and included an opportunity for the public to provide input and comment, both in writing as well as orally.

Pursuant to the *California Code of Regulations*, Title 5, Section 9511, the SBE appoints the CFCC. The CFCC is the advisory committee of 20 mathematics practitioners, the majority of whom are mathematics instructors at the time of appointment, “to assist in the process of developing a curriculum framework.”

The Mathematics CFCC met a total of eight times in 2020 to review and provide edits and recommendations on the draft *Mathematics Framework* chapters in accordance with the SBE-established guidelines. It then recommended a draft to the IQC for further review and action.

During the meeting of the IQC held on May 19 and 20, 2021, the IQC reviewed public comments from the first 60-day public comment period ending on April 8, 2021. At the meeting, the IQC considered and made numerous revisions, including adding an appendix that includes learning progressions, adding guidance on providing adequately-challenging instructional opportunities and differentiation, removing some references that did not best reflect the goals of the framework, and providing additional information regarding the specific needs of various students.

Further, the IQC discussions underscored that decisions about course acceleration, as well as honors and Advanced Placement (AP) courses, are local ones and requested that the next draft include specific guidance on acceleration (including middle school acceleration) and serving high achievers as well as students with greater needs.

The *Mathematics Framework* revisions recommended in 2021 were meant to address ways curriculum can meet the needs of as many students as possible, making math more accessible and to provide research and guidance so schools can make the best local decisions that provide options and improve math outcomes for all students.

The IQC acted to recommend the *Mathematics Framework* to the SBE, following the completion of the edits and another 60-day public comment period. The second 60-day public comment period took place between March 14, 2022, and May 16, 2022.

The second 60-day review period resulted in a large volume of comments, including many detailed, actionable suggestions for edits to the draft framework. In response to this, California Department of Education (CDE) and SBE staff worked with the Region 15 Comprehensive Center at WestEd (R15CC) to review the draft and the public comments and suggested additional edits. During the rest of 2022 and early 2023, the R15CC worked with SBE and CDE staff to prepare a draft reflecting proposed edits and changes to the second 60-day review draft framework for the SBE's consideration. This draft addresses many of the critiques and suggestions for improvement made by the public during the framework revision process.

The revised draft reflects hundreds of individual line edits throughout all of the chapters responsive to hundreds of specific public suggestions fielded during the second 60-day public comment period. It also includes structural and content changes responsive to more general public comments, direction from the IQC, and the SBE-adopted framework guidelines. The public comments generally underscored the importance of clear phrasing and reader-friendly structuring and formatting of information throughout the framework. Recognizing this, edits intended to improve the text's clarity and conciseness, and to improve graphics, were made to each chapter. In some places text was rearranged, subheads or paragraph heads were added, and/or transitions were created to improve reader friendliness. To improve the flow and readability of the text, all longer vignettes were moved to an appendix, with hyperlinks. To support framework-wide cohesion, discussion of the instructional design approach, which recurs across chapters, was clarified and standardized.

For information regarding the timeline, see the *Mathematics Framework* web page.

What are the core ideas of the new *Mathematics Framework*?

The framework offers guidance for enacting the standards through curriculum and instructional approaches grounded in research and reflecting best practices across the globe. It emphasizes development of the habits of mind and habits of interaction described in the Standards for Mathematical Practice—for example, persevering in problem solving, explaining one's thinking, constructing arguments—that students need in order to become powerful users of mathematics and better interpret and understand their world. For this reason, this framework highlights examples that support multidimensional learning.

The use of open, authentic, multidimensional tasks includes, but is not limited to, learning

mathematical ideas not only through numbers, but also through words, visuals, models, algorithms, multiple representations, tables, and graphs; from moving and touching; and from other representations. Studies show that when learning reflects the use of two or more of these means, the learning experience improves for all kinds of learners. These approaches align with the principles of Universal Design for Learning, a framework designed to make learning more accessible and inclusive.

The framework also builds on learning science research showing that students learn best when they understand how major concepts are connected to one another and to real-world examples and applications. With the goal of motivating students to learn coherent, focused, and rigorous mathematics, this framework identifies three Drivers of Investigation (DIs), which provide the "why" of learning mathematics, to pair with four categories of Content Connections (CCs), which provide the "how and what" of the mathematics that is to be learned in an activity. Together with the Standards for Mathematical Practice, the Drivers of Investigation are meant to propel the learning of the ideas and actions framed in the Content Connections in ways that are coherent, focused, and rigorous.

- Big Ideas
  - In each grade band section (TK–2, 3–5, 6–8, 9–12), the description focuses on several Big Ideas that have great impact on students' conceptual understanding, and which encompass multiple content standards.

- Drivers of Investigation
  - DI1: Make Sense of the World (Understand and Explain)
  - DI2: Predict What Could Happen (Predict)
  - DI3: Impact the Future (Affect)

- Content Connections
  - CC1: Reasoning with Data
  - CC2: Exploring Changing Quantities
  - CC3: Taking Wholes Apart, Putting Parts Together
  - CC4: Discovering Shape and Space

How does California's framework approach reflect what is happening in high-achieving countries in mathematics?

The US ranks well below the average on international tests of mathematics achievement, and California ranks well below the average within the US. The framework builds on the strategies used in a number of the highest-achieving jurisdictions (e.g., Estonia, Finland, Japan, and South Korea) that pursue an integrated curriculum—connecting the domains of mathematics with one another as students collaborate in using data to solve real-world problems. These countries pursue a common curriculum in elementary and middle school, supporting more students in

reaching higher level mathematics. The framework illustrates how this integrated approach with many different kinds of supports can be used to expand the number of students excelling in mathematics and heading for science, technology, engineering, and mathematics (STEM) careers.

What is the meaning of the word "Equity" in the draft *Mathematics Framework*?

Chapter 1 of the framework addresses this question: "One aim of this framework is to respond to the structural barriers to mathematics success. Equity—of access and opportunity—is essential and influences all aspects of this document. Overarching principles that guide work towards equity in mathematics include the following:

- All students deserve powerful mathematics instruction. High-level mathematics achievement is not dependent on rare natural gifts, but rather can be cultivated.
- All students, regardless of background, language of origin, learning differences, or foundational knowledge are capable and deserving of depth of understanding and engagement in rich mathematics tasks.
- Student engagement must be a goal in designing mathematics curriculum, alongside critical content goals.
- Students' cultural backgrounds, experiences, and languages are resources for teaching and learning mathematics.

Does the draft *Mathematics Framework* provide guidance for a range of students including students identified as high achieving or gifted?

By direction of the IQC at its May 2021 meeting, the revised draft of the *Mathematics Framework* includes specific guidance on acceleration (including middle school acceleration) and serving advanced students.

Since the development of the 2013 *Mathematics Framework*, new research has emerged that can be used to inform local conversations about how to best serve high achieving students. Throughout the chapters, the framework provides guidance about multiple strategies, including and in addition to acceleration, for ensuring that students have the opportunity to reach advanced mathematics courses, such as calculus and statistics, during high school. Additionally, it discusses the conditions research has found to be associated with successful acceleration, including teacher training.

What does the draft *Mathematics Framework* say about middle school mathematics acceleration programs?

The IQC discussions from the May 2021 meeting underscored that the decision about

acceleration/honors and AP courses is a local one and requested that the updated draft include specific guidance on acceleration (including middle school acceleration) and serving high achievers and gifted students.

Chapter 8 of the draft *Mathematics Framework* notes that: “Some students will be ready to accelerate into Algebra I or Mathematics I in eighth grade, and, where they are ready to do so successfully, this can support greater access to a broader range of advanced courses for them.”

The framework also notes that successful acceleration requires a strong mathematical foundation, and that earlier state requirements that all students take eighth grade Algebra I were not implemented in a manner that proved optimal for all students. It cites research about successful middle school acceleration leading to positive outcomes for achievement and mathematics coursetaking, built on an overhaul of the middle school curriculum to prepare students for Mathematics I in eighth grade, teacher professional development and collaborative planning time, and an extra lab class for any students wanting more help.

To support successful acceleration, the framework also urges, in chapter 8: “For schools that offer an eighth grade Algebra course or a Mathematics I course as an option in lieu of Common Core Math 8, both careful plans for instruction that links to students’ prior course taking and an assessment of readiness should be considered. Such an assessment might be coupled with supplementary or summer courses that provide the kind of support for readiness that Bob Moses’ Algebra project has provided for many years for underrepresented students tackling Algebra.”

For students who do not accelerate, the draft *Mathematics Framework* emphasizes the importance of deep treatment of the topics laid out in the CA CCSSM and notes that the CA CCSSM are significantly more rigorous than those from previous grade eight content standards. They address the foundations of algebra and geometry by including content that was previously part of the Algebra I course, including but not limited to a more in-depth study of linear relationships and equations, a more formal treatment of functions, and the exploration of irrational numbers. Thus, students who complete the CA CCSSM courses for grades six, seven, and eight are also better prepared than in the past for more challenging mathematics courses in high school.

What does the draft *Mathematics Framework* say about the high school curriculum?

To align with SBE guidelines and the CA CCSSM, the framework affirms two high school pathways—traditional and integrated—and clarifies the autonomy of local schools and districts in designing courses and pathways that best meet the needs of local families. The guidance helps educators to integrate and align math concepts taught at the middle and high school levels, ensure that all high school math pathways are open to all students, and expand high school math course options to encourage more students to go beyond minimum course-taking requirements.

The high school pathway discussion and graphic in chapter 8 of the draft *Mathematics Framework* more clearly articulate course sequencing and course options, including pathways to pre-calculus

and calculus, as well as options for financial literacy, data science, statistics, and discrete math. The pathways figure was updated to clarify that students should be able to choose from more course offerings during their third- and fourth-year math classes and shift from non-STEM to STEM pathways or vice versa.

The draft *Mathematics Framework* does not eliminate the ability of local educational agencies to develop course pathways that differ from these options. It also provides information about how colleges are considering different mathematics pathways, including the University of California and California State University systems' Statement of Competencies in Mathematics Expected for Entering College Students. (See Statement of Competencies in Mathematics Expected for Entering College Students (PDF).) And it includes advice about the specific competencies' students will likely need to succeed in STEM pathways in college. (See chapter 8 and appendix A.)

What does the draft *Mathematics Framework* say about access to calculus in high school?

The draft *Mathematics Framework* includes calculus in the possible high school pathways and also suggests ways to enable more students to get access to calculus. It notes that many high schools currently organize their coursework in a manner that requires eighth grade acceleration in order to reach calculus or other advanced mathematics courses by senior year. While some students succeed with this approach, acceleration has proved a problematic option for other students who could reach higher level math courses but would benefit from a stronger foundation in middle school mathematics.

The draft framework offers evidence about what approaches can support successful acceleration for more students. At the same time, it recognizes that a strong foundation is most important for later success, quoting the University of California's board of admissions that "strongly urges students not to race to calculus at the cost of full mastery of the earlier math curriculum…. A strong grasp of these ideas is crucial for college coursework in many fields, and students should be sure to take enough time to master the material. Choosing an individually appropriate course of study is far more important than rushing into advanced classes without first solidifying conceptual knowledge."

Accordingly, the draft framework offers a variety of options for reaching calculus in high school both with and without eighth grade acceleration. Among these, it notes in chapter 8:

> One consideration in sequencing mathematics courses is the desire to enable students who would like to reach Calculus by the end of high school to do so. Currently, most high schools require courses in Algebra, Geometry, Algebra II, and Pre-calculus before taking a course in Calculus, or a pathway of Mathematics I, II, III, then Precalculus. This sequence means that students cannot easily reach Calculus in high school unless they have taken a high school algebra course or Mathematics I in middle school.

An alternative to eighth grade acceleration would be to adjust the high school curriculum instead, eliminating redundancies in the content of current courses, so that students do not need four courses before Calculus. As enacted, Algebra II tends to repeat a significant amount of the content of Algebra I, and Precalculus repeats content from Algebra II. While recognizing that some repetition of content has value, further analysis should be conducted to evaluate how high school course pathways may be redesigned to create more streamlined pathways that allow students to take three years of middle school foundations and still reach advanced mathematics courses such as calculus.

The Mathematical Association of America (MAA) and the National Council of Teachers of Mathematics (NCTM) issued a joint statement that included the premise: “A calculus course can provide students with important foundations for a variety of further studies, particularly in mathematically intensive fields. To ensure that calculus fulfills its multiple purposes, high schools and colleges should design curriculum and instruction in ways known to promote student success. A high school calculus course should not be the singular end goal of the PK–12 mathematics curriculum at the expense of providing a broad spectrum of mathematical preparation. All calculus courses should focus on proficiencies that enable students to succeed in the wider world of mathematical studies.” (See MAA and NCTM Joint Statement).

What is the role of Data Science in the draft *Mathematics Framework*?

In response to the explosion of data use in our daily lives and across fields, the framework highlights the opportunities data present for bringing mathematics alive for students. It discusses ways to integrate data use into math instruction throughout the grades and to encourage students to become proficient at understanding and using data—including helping students identify misleading data and use data to make decisions in their roles as global citizens.

Chapter 5: *Mathematical Foundations for Data Science* recognizes that data science has become tremendously important in the field since the last framework. The framework also documents the recent decisions of several high-achieving countries to include more data science in their curricula, as developing the ability to interpret, use, and analyze data has become a worldwide concern.

What does the draft *Mathematics Framework* say about “tracking”?

The framework presents national and international evidence suggesting that early tracking decisions which differentiate content before secondary school in ways that later prevent access to advanced courses for many students ultimately tend to lower achievement overall and are not necessary when productive teaching strategies are used. It also recognizes that thoughtful grouping strategies that support students in various ways for tutoring and small group work, as well as supplemental coursework and later course taking, can be helpful when they do not close off options for ultimate access to higher-level mathematics.

The draft states in chapter 9: “This framework proposes grouping systems and other supports that keep higher level pathways open to more students for a longer time, while enabling high-achieving students to move more rapidly and deeply through content, as appropriate. The framework recognizes the diversity of student achievement and sets out ways to teach that ensure that all students receive appropriate support and challenge—including providing all students with challenging work rather than leaving some students bored or working at levels lower than what they may be capable of, which can happen if teachers require the entire class to stay together or learn the same content in the same way or at the same pace.”

The framework notes in chapter 9: “The remainder of this chapter sets out the different ways students may be challenged and supported in mathematics classes with examples of how districts and schools have enacted systems of grouping that support a wider range of students in accessing higher level content. If the goal is to open mathematics pathways to more students and give greater challenge to high achieving students to develop broader proficiency and long-term interest in quantitative fields, then this framework recommends reshaping the content that is offered to students—the way it is taught, and the organization of students learning the content—in the following ways.” It goes on to address, with research and examples, teaching multidimensional mathematics through big ideas and connections; personalized learning, including tutoring; additional classes; and flexible versions of student grouping.

The framework notes that: “While early tracking of students into low-level courses has been problematic, there is evidence that thoughtful grouping of students to ensure they receive high-quality instruction geared to their needs at a moment in time can be helpful. Such an approach can help students who need to fill gaps in their prior learning as well as high-achieving students who are ready for greater challenges.” The framework reminds educators of the California Mathematics Placement Act of 2015, which requires that every high school placement policy of a local educational agency meet the following requirements (CDE, 2016):

- Systematically takes multiple objective academic measures of pupil performance into consideration.
- Includes at least one placement checkpoint within the first month of the school year to ensure accurate placement and to permit reevaluation of individual student progress.
- Requires an annual examination of pupil placement data to ensure that students are not held back in a disproportionate manner on the basis of their race, ethnicity, gender, or socioeconomic background.
- Requires a report on the results of the annual examination by the local educational agency to its governing board or body.
- Offers clear and timely recourse for each pupil and his or her parent or legal guardian who

questions the student's placement.

- For non-unified school districts, addresses the consistency of placement policies between elementary and high school districts.

What guidance does the framework provide on learning recovery for students who need to catch up?

The framework acknowledges that students develop and learn at different times and rates. Students may arrive in new grade levels with unfinished learning from earlier levels. Schools should not automatically assume these students to be low achievers requiring placement in a separate class or track that will not reach grade level standards. Rather, instructional decisions should be grounded in California's Multi-Tiered System of Support (MTSS), which is designed to provide the means to quickly identify and meet the needs of all students through instructional supports within and beyond the classroom. Additional information is available on the MTSS web page.

The framework offers three strategies to support learning recovery for students: 1) Providing additional support and expanded learning time—e.g., in the form of courses or support time scheduled alongside their primary math class or, during high school, taking more than one mathematics course a year (including summer programming) to continue progress toward more advanced courses; 2) providing personalized learning—that is, learning experiences customized "for each student according to his or her unique skills, abilities, preferences, background, and experiences" (Herold, 2019); and 3) including one-on-one or small group tutoring. Provided regularly, and systematically, such tutoring can result in substantial gains in mathematics achievement and allow students to accelerate their learning and sustain a path to higher level courses. It can also reduce the felt need for lower-track classes that derail students at an early age from paths leading to potential STEM careers.

How are the needs of English learners addressed in the framework?

Students who are English learners integrated in an English-only setting face the challenge of learning mathematics content and the language of instruction simultaneously. These students bring experiences, perspectives, and ideas that enrich the classroom for all, and instructional strategies that are designed to meet their needs support mathematical learning for all students. English learners are most supported in learning the languages of English and mathematics when they are given the opportunity to use multiple tools for learning about mathematics, including visual tools that represent problems graphically as well as verbally, to reason about mathematics in small-group and whole-class discussions, and to look at, listen to and connect with the ideas of other students (Zwiers, 2018). The framework also draws on research that illustrates how, in addition to making work visual, teachers can strategically group students for language development and provide opportunities for pre-learning (Darling, 2019).

Accordingly, the framework supports instruction for linguistically and culturally diverse English

learners who are developing mathematical proficiency rooted in and informed by the California English Language Development Standards (CA ELD Standards) and the California Department of Education's advice for integrating the CA ELD Standards into mathematics teaching (CDE, 2021a). Knowledge of and alignment with the CA ELD Standards offers mathematics educators ways to strengthen instructional support that benefits all students. Building comprehensive mathematics instruction based on an understanding of individual CA ELD Standards ensures that learning reflects a meaningful and relevant use of language that is appropriate to grade level, content area, topic, purpose, audience, and text type.

What guidance does the framework provide in approaches to instruction for students with disabilities?

The framework asserts that the range of student backgrounds, learning differences, and perspectives, taken collectively, are an instructional asset that can be used to launch and support all students in a deep and shared exploration of the same context and task.

All students, including those with learning differences, benefit from instruction that uses open tasks and that allows for peer interaction and support, multiple approaches, and multiple means of representing their thinking. Students diagnosed with mathematics learning disabilities additionally benefit from small group instruction and one-to-one tutoring that teaches cognitive strategies as well as mathematics facts. Instructional decisions should be grounded in California's Multi-Tiered System of Support (MTSS), which is designed to provide the means to quickly identify and meet the needs of all students. Additional information is available on the MTSS web page.

CDE's Clearinghouse for Specialized Media and Technology also provides instructional resources in accessible and meaningful formats to students with learning differences and identified disabilities, including students who have hearing or vision impairments, severe orthopedic impairments, or other print disabilities.

Questions: Curriculum Frameworks and Instructional Resources Division | CFIRD@cde.ca.gov | 916-319-0881

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